Monetary Macroeconomics

Chapter 7:
Capital

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Including Other Assets in Our OLG Model

Fiat money is not the only asset available in the real world for saving. Now we will analyze how the presence of other assets affect the demand for money.

Chapter 7: Capital

Capital is an asset that can be used for production, e.g. machinery, seeds, etc.

We generally denote the production function with $F(K, L)$ where K is capital, and L is labor.

One usually assumes that each period capital depreciates at rate δ, such that $0 \leq \delta \leq 1$.

So next period one still has $(1 - \delta)K$ capital left after receiving the output $F(K, L)$.

The Model

Consider our OLG model with 2 period lived agents. There is one country, and no fiat money!

The young have y units of endowment that can be either consumed or invested as capital. The old have no endowment.

Assume that when k units are invested in the production technology then $f(k)$ units of the consumption good are produced next period.

Properties of the Production Function

In general $f'(k) > 0$ and $f''(k) < 0$, so that we have diminishing marginal product of capital.

That is the added output from an extra unit of capital gets smaller as capital increases.

Assume that the initial old have had k_0 units of capital that produced $f(k_0)$ goods in the first period.

We assume that capital depreciates completely after one period. Thus $\delta = 1$.

Consider stationary allocations.

Generation t's budget constraints:

when young: $c_1 + k \leq y$.
when old: $c_2 \leq f(k)$.

Hence the lifetime budget constraint is:

$$c_2 \leq f(y - c_1)$$

So his problem is

$$\begin{aligned} \max \quad & u(c_1, c_2) \\ \text{s.t.} \quad & c_2 \leq f(y - c_1) \end{aligned}$$

The Lagrangian is

$$\mathcal{L} = u(c_1, c_2) + \lambda\left[f(y - c_1) - c_2\right]$$

$$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{\partial u(c_1, c_2)}{\partial c_1} + \lambda[f'(y - c_1)(-1)] = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = \frac{\partial u(c_1, c_2)}{\partial c_2} + \lambda(-1) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = f(y - c_1) - c_2 = 0$$

So

marginal utility of $c_1 = \lambda *$ marginal product

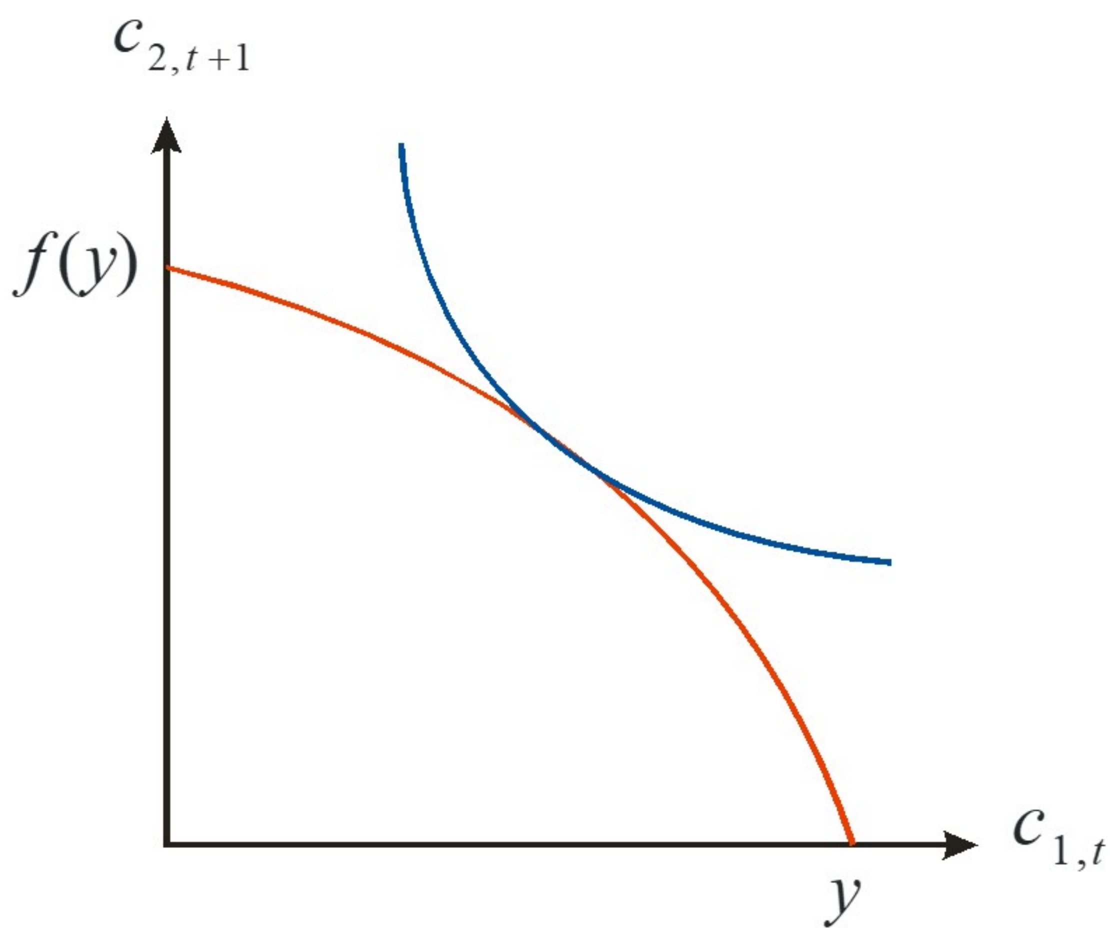
marginal utility of $c_2 = \lambda$

Thus

$$-\text{MRS} = \text{MP}$$

At the solution (c_1^*, c_2^*), the marginal rate of substitution between the two period consumptions is equal to the marginal product of capital.

Note that there is nothing special about the OLG model here. Agents are not trading with each other.


$c_{2,t+1}$
$f(y)$
$c_{1,t}$
y

A Model of Private Debt

Here we will find conditions so that more than one asset can exist in the economy.

Consider an OLG model with 2-period lived consumers. There is no fiat money in the economy.

Assume first that there is no capital either. There is one type of consumption good that everybody likes to consume when young and when old.

There are two types of people: borrowers and lenders.

Borrowers have no endowment when young and y units of the consumption good when old.

Lenders have y units of the consumption good when young and nothing when old.

In this model agents can issue IOUs (I Owe You).

Let r be the gross real rate of interest on these private loans.

The Borrower's Problem

$$\begin{aligned} \max \quad & u(c_1, c_2) \\ \text{s.t.} \quad & c_1 \leq b \\ & c_2 + rb \leq y \end{aligned}$$

where b is the amount he borrows when young.

Equivalently,

$$\begin{aligned} \max \quad & u(c_1, c_2) \\ \text{s.t.} \quad & c_1 + \frac{c_2}{r} \leq \frac{y}{r} \end{aligned}$$

Problem: Draw this problem in a graph. Denote the solution. What is the impact of a decrease in r on the solution?

Demand for Loans

The slope of the budget line of the borrower gets flatter as r decreases. Hence the solution to his problem will move to the right. That means that c_1^* increases.

Thus b^* will also increase as r decreases for this individual.

Therefore, generally the aggregate demand for loans decreases as r increases. The demand has a negative slope.

Lender's Problem

$$\begin{aligned} \max \quad & u(c_1, c_2) \\ \text{s.t.} \quad & c_1 + l \leq y \\ & c_2 \leq rl \end{aligned}$$

where l is the amount he lends to the other type when young.

Equivalently,

$$\begin{aligned} \max \quad & u(c_1, c_2) \\ \text{s.t.} \quad & c_1 + \frac{c_2}{r} \leq y \end{aligned}$$

Problem: Draw this problem in a graph. Denote the solution. What is the impact of an increase in r on the solution?

Supply of Loans

As r increases, the slope of the budget line of the lender will get steeper. Thus the solution to his problem will generally move to the left. Thus c_1^* will decrease.

Hence l^* will increase!

Therefore, generally the supply of loans increases as r increases. The supply has a positive slope.

Equilibrium Interest Rate

The equilibrium will be where the supply of loans and demand for loans intersect. So the equilibrium interest rate will be such that $b^* = l^*$. Denote that interest rate with r^*.

Include Capital into the Model

Now suppose that there is also another asset available in the economy, called capital, that pays gross rate of return x.

Note: A possible production function could be $f(k) = xk$.

That is the lenders now have two options to save: invest in capital or lend to the borrowers.

What is the Relationship Between r^* and x?

When $x < r^*$:

No lender would invest in capital. They would give all their excess goods $(y - c_1^*)$ to borrowers.

When $x > r^*$:

Lenders would ask x as interest rate from the borrowers. Otherwise they would not lend them (there is a better option, capital pays more!).

But at x, borrowers demand only little amounts of loans — supply exceeds demand. So lenders would hold a *mixed portfolio* of capital and loans.

Rate of Return Equality

If individuals are willing to hold all of the available assets simultaneously, the rates of return on these assets must be identical.

Can Fiat Money Coexist with Other Assets?

Now assume that there is also fiat money in this economy.

The prices satisfy

$$\frac{p_t}{p_{t+1}} = \frac{n}{z}$$

where n is the population growth rate and z is the money supply growth rate.

Then a lender would be holding money and capital together *only if*

$$\frac{n}{z} = x$$

That is when the rate of return on fiat money is equal to the rate of return on capital (in this model also the marginal product of capital).

If $\frac{n}{z} < x$, then money will not be valued in the economy.

If $\frac{n}{z} > x$ then lenders would not invest in capital.

This is because the two assets are perfect substitutes in this model!

Risk

Sometimes lending can be very risky — there is possibility of default.

Then the *rate of return equality* requires that

the *expected value* of the rate of return on the risky asset equals to the rate of return on the safe asset

to be satisfied in equilibrium if both assets are held by *risk-neutral* individuals simultaneously.

However, many people are risk-averse in the real world. So they demand a higher expected value of the rate of return on a risky asset in order to hold it. That is called a *risk premium*.

Thus in the real world fiat money can coexist with other assets that pay a higher rate of return since they would *not* be perfect substitutes!

Relationship between Interest Rates and Inflation

Denote R_t the gross nominal interest rate. So for every CHF invested, you get back CHF R_t next period.

Denote r_t the gross real interest rate. So for every good invested, you get back r_t units of good next period.

In a stationary monetary equilibrium it must be the case that

$$r_t = R_t \frac{p_t}{p_{t+1}} = R_t \frac{n_t}{z_t}$$

So $R_t = \frac{z_t}{n_t} r_t$.

One can show that

$$R_t - 1 \approx (r_t - 1) + \left(\frac{z_t}{n_t} - 1\right)$$

That is the net nominal interest rate is approximately equal to the sum of net real interest rate and the net inflation rate.

Since usually the inflation rate is *not known*, but only anticipated, one substitutes the *expected* inflation in the formula (known as the **Fischer Equation**).

How Do the Interest Rates React to Anticipated Inflation?

The Fisher Effect: The full adjustment of the nominal interest rate to anticipated inflation.

That is, if there is an anticipated change in the inflation rate, then the nominal interest rate fully adjusts and the real interest rate remains unchanged.

The Tobin Effect: There will be a substitution of capital for money in reaction to an increase in anticipated inflation.

That is, if there is an anticipated increase in inflation, then consumers will decrease their fiat money holdings and hence they will invest more in capital. Thus output will increase. But if the production function has diminishing marginal product, then the rate of return on capital will decrease. Thus the real interest rate will decrease. And the nominal interest rate will not adjust fully.