Monetary Macroeconomics

Lecture 2:

A Simple Model of Fiat Money

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Review

Questions we would like to answer:

- What is the role of money in the economy?
- Why do people use money?

We started using an overlapping generations model (OLG) to answer these questions.

Recall the Model

Agents live for two period. There is one perishable consumption good.

Agents have y units of endowment of the consumption good when young, and 0 units when old.

They like to consume in both periods.

We wrote the **feasibility condition** in its most general form as

$$N_t c_t^t + N_{t-1} c_t^{t-1} \leq N_t y$$

A competitive equilibrium is prices and an allocation such that given prices, the allocation solves the agents' maximization problems, and the allocation is feasible.

We have seen that **the unique competitive equilibrium** in this economy is *autarky*. Hence no trade can take place, and the agents consume their whole endowment when young and nothing when old.

The golden rule allocation in a stationary economy with *constant population* solves the problem

$$\max u(c_1, c_2)$$
$$\text{subject to } c_1 + c_2 \leq y$$

Note that the golden rule allocation cannot be attained in a competitive equilibrium.

Fiat money

Assume that there is a good that can be stored costlessly by the agents but it cannot be produced or consumed. Agents can exchange it but it is *intrinsically useless*. It is not perishable!

We call this **fiat money**.

A **monetary equilibrium** is a competitive equilibrium where fiat money is valued.

That is the agents trade fiat money for consumption good.

Note: For fiat money to be valued, the money supply must be limited and it must be impossible to counterfeit.

Introduce Fiat Money to this Model

At period t, generation t is born with y units of initial endowment of the consumption good and no fiat money.

He can trade some of his endowment for money so that he can buy some consumption good when he is old.

His budget constraint in period t in money terms is

$$p_t c_t^t + m_t \leq p_t y$$

The first term is the monetary value of his consumption when young, m_t is the money he acquires when young, and the right hand side is the monetary value of his endowment when young.

His budget constraint next period $(t+1)$ in money terms is

$$p_{t+1}c^t_{t+1} \leq m_t$$

The left hand side is the monetary value of his consumption when old, and the right hand side is the money he acquired last period.

We can combine the two budget constraints and obtain the *life-time budget constraint* of generation t.

$$p_t c^t_t + p_{t+1}c^t_{t+1} \leq p_t y$$

The left hand side is the value of his total (lifetime) consumption and the right hand side is his initial (lifetime) wealth.

Lifetime Budget Constraint and Prices

$$p_t c_t^t + p_{t+1} c_{t+1}^t \leq p_t y$$

$$c_t^t + \frac{p_{t+1}}{p_t} c_{t+1}^t \leq y$$

Recall that in a competitive equilibrium the prices are given — the agents do not have any control on prices.

Denote

$$\frac{p_{t+1}}{p_t} = 1 + \pi_t$$

π is the (net) inflation rate.

How do We Find the Prices?

Assume that there is M_t units of fiat money in the economy in period t.

In equilibrium money supply must equal to money demand!

Who demands money in this economy? The young only.

How much?

From his budget constraint when young

$$m_t = p_t y - p_t c_t^t$$

So the total money demand is

$$M_t = (p_t y - p_t c_t^t) N_t$$

Thus

$$M_t = p_t N_t (y - c_t^t)$$

and the price level is given by

$$p_t = \frac{M_t}{N_t (y - c_t^t)}$$

$$p_{t+1} = \frac{M_{t+1}}{N_{t+1} (y - c_{t+1}^{t+1})}$$

Therefore the rate at which the price changes is

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{N_{t+1} (y - c_{t+1}^{t+1})} \frac{N_t (y - c_t^t)}{M_t}$$

Stationary Monetary Equilibrium with Constant Population

Assume that $N_t = N$ for all t.

Consider the stationary allocations.

So $c_t^t = c_1$ and $c_{t+1}^t = c_2$ for all t.

Thus

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{N_{t+1}(y - c_{t+1}^{t+1})} \frac{N_t(y - c_t^t)}{M_t}$$

becomes

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{M_t}$$

When the Money Supply is Constant

Then

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{M_t} = 1$$

That is the prices are also *constant*!

Then the generation t's budget constraint reduces from

$$c_t^t + \frac{p_{t+1}}{p_t} c_{t+1}^t \leq y$$

to

$$c_1 + c_2 \leq y$$

So his maximization problem is

$$\max u(c_1, c_2)$$
$$\text{subject to } c_1 + c_2 \leq y$$

The generation t's maximization problem *is identical* to the social planner's problem that maximizes future generations' utility.

The stationary monetary equilibrium is the golden rule allocation!

By introducing fiat money in the economy the welfare of the individuals in the economy are improved.

The competitive equilibrium is not autarky anymore. The young and the old can trade with each other and everyone is better off!

We stopped here last time.

Note: Generally the competitive equilibrium is not the golden rule allocation. This is true only in the case of a stationary equilibrium.

However, after introducing fiat money into the economy, everyone is still better off than before because people can trade now.

What Changes if the Population is Growing and the Money Supply is Constant?

Assume $N_t = nN_{t-1}$ for all t where $n > 1$.

And $M_t = M$ for all t.

Then

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{N_{t+1}(y - c_{t+1}^{t+1})} \frac{N_t(y - c_t^t)}{M_t} = \frac{1}{n}$$

in a stationary equilibrium.

Hence the budget constraint of an agent born in t can be given as

$$c_1 + \frac{1}{n}c_2 \leq y$$

So his maximization problem is

$$\begin{aligned} &\text{max } u(c_1, c_2) \\ &\text{subject to } c_1 + \frac{1}{n}c_2 \leq y \end{aligned}$$

How About the Social Planner's Problem?

The feasibility constraint with a growing population is

$$N_t c_1 + N_{t-1} c_2 \leq N_t y$$
$$n N_{t-1} c_1 + N_{t-1} c_2 \leq n N_{t-1} y$$
$$c_1 + \frac{1}{n} c_2 \leq y$$

Hence the social planner's problem is

$$\text{max } u(c_1, c_2)$$
$$\text{subject to } c_1 + \frac{1}{n} c_2 \leq y$$

The social planner's problem is identical to the agent's utility maximization problem.

Hence the stationary monetary equilibrium is again the golden rule allocation!

Problem:

Graph the stationary monetary equilibrium and the golden rule allocation when the population is growing and when the money supply is constant.

Summary of the Findings

We have shown that when the money supply is constant, the stationary monetary equilibrium is the golden rule allocation both when the population is constant and when the population is growing.

Role of Fiat Money

With the introduction of fiat money, allocations that were not attainable before can be achieved now in equilibrium.

	Stationary	Other equilibria
No Money	Autarky	Autarky
Fiat Money	Golden Rule	Equilibria with Trade

Fiat money expands the set of attainable allocations in equilibrium!

Notation

The textbook uses the variable $v_t = \frac{1}{p_t}$, the **value of fiat money** in terms of the consumption good.

Hence $\frac{v_{t+1}}{v_t} = \frac{p_t}{p_{t+1}}$ is the real rate of return on fiat money.

This gives how many units of consumption good can be obtained in period $t+1$ if one unit was sold for money in period t.

q_t denotes a young person's **real demand for money**. It is how many units of consumption good he is willing to sell in exchange for fiat money.

$$q_t = \frac{m_t}{p_t} = \frac{p_t(y - c_t^t)}{p_t} = y - c_t^t$$

Problem

Consider the overlapping generations model with fiat money.

Assume that the utility of an agent born in t is given by

$$u(c_t^t, c_{t+1}^t) = \ln c_t^t + \beta \ln c_{t+1}^t$$

where c_t^t is consumption when young, c_{t+1}^t is consumption when old, and $0 < \beta < 1$ is the period discount factor.

Show that the real money demand of this agent is equal to

$$q_t = \frac{\beta y}{1 + \beta}$$

Quantity Equation

The Quantity Equation is

$$pY = vM$$

where pY is the nominal output, v is the velocity of money, and M is the money supply.

It does not have a theoretical base on its own.

v shows how many times money has to circulate within a year when the nominal output is pY.

Quantity Theory of Money

In the Quantity Theory of Money, the velocity is assumed to be constant, and that the real output is assumed not to depend on nominal money supply.

$$pY = \bar{v}M$$

The Quantity Theory of Money says that the *price level is exactly proportional to the quantity of money in the economy*.

Recall that in our model

$$p_t = \frac{M_t}{N_t(y - c_t^t)}$$

So when M_t is doubled, then p_t doubles, too.

This model is consistent with the Quantity Theory of Money.

Neutrality of Money

The nominal size of fiat money stock has no effect on the real values of consumption or money demand in equilibrium.

In our model the agents' real demand for money does not depend on the fiat money stock, but depends on the rate of return on money $\left(\frac{p_{t+1}}{p_t}\right)$. Their utility maximization problem does not involve the fiat money stock.

Relationship between Money Supply, Prices, and Output

A central bank's major instrument of monetary policy is the growth rate of money supply, targeted either directly or indirectly through some nominal target like an interest rate (more on that later during the semester). Different central banks choose to adjust different definitions of money.

So they should know the relationships between prices, money supply, and output.

In particular, *what is the effect of changes in money growth on the rate of inflation or output growth*?

The following article analyzes long-run macroeconomic data on prices, money, and output, for a large set of countries.

"Some Monetary Facts" by George McCandless and Warren Weber (1995), Quarterly Review, Vol.19, No.3, Federal Reserve Bank of Minneapolis

https://www.minneapolisfed.org/research/quarterly-review/some-monetary-facts

The long-run data show:

- Growth rates of the money supply and the general price level are highly correlated for all three money definitions, for the full sample of countries, and for both subsamples.

  This correlation coefficient is close to 1.

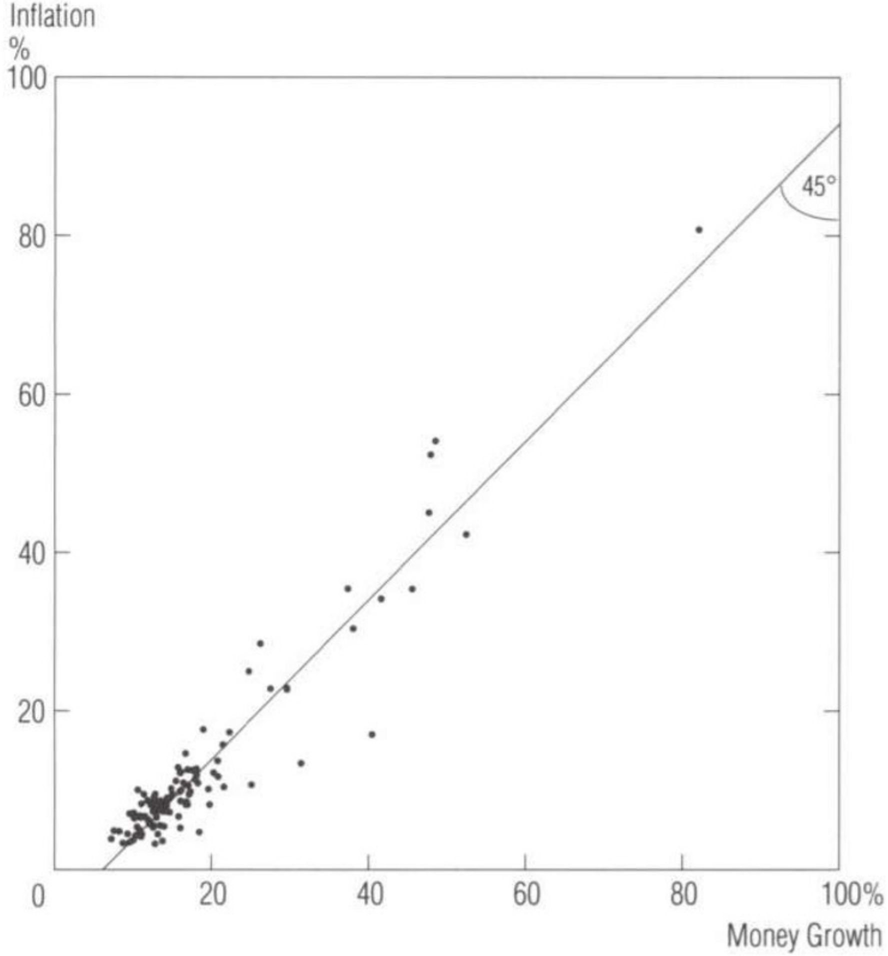
- The growth rates of money and real output are not correlated, except for a subsample of countries in the Organization for Economic Co-operation and Development (OECD), where these growth rates are positively correlated.

- The rate of inflation and the growth rate of real output are essentially uncorrelated.

Source: McCandless and Weber (1995)

Money Growth and Inflation: A High, Positive Correlation

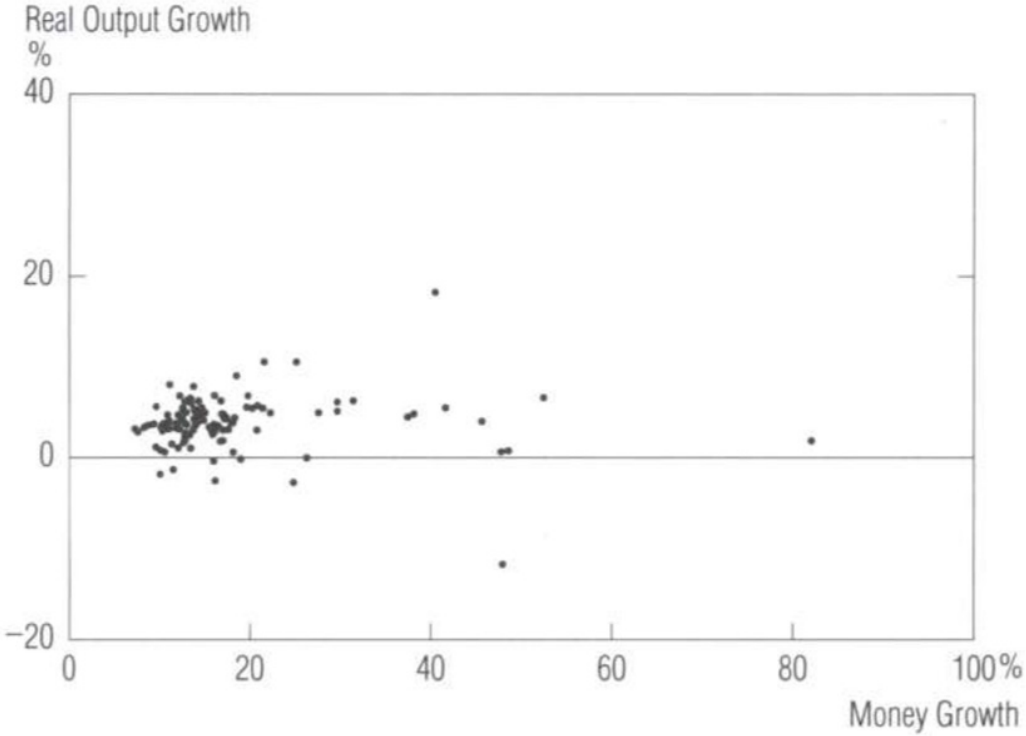
Average Annual Rates of Growth in M2 and in Consumer Prices During 1960–90 in 110 Countries



Source: International Monetary Fund

Money and Real Output Growth: No Correlation in the Full Sample . . .

Average Annual Rates of Growth in M2 and in Nominal Gross Domestic Product, Deflated by Consumer Prices During 1960–90 in 110 Countries



Source: International Monetary Fund

Lessons learned:

- We can adjust long-run inflation by adjusting the growth rate of money – though we cannot necessarily hit specific inflation targets!

- Monetary policy has no long-run effects on real output – of course, this does not rule out that there might be short-run effects.

Note: If the long-run effects of monetary policy on real economic activity is truly zero, then any short-run successes in reducing downturns can only come about at the expense of reducing up-turns!

A more recent article revisited these monetary facts:

"Monetary Facts Revisited" by Pavel Gertler and Boris Hofmann (2016), BIS Working Paper, No 566

- There is a significant long-run link between money growth and inflation, but it has significantly weakened over time, particularly in advanced economies.

- Credit growth has signicant predictive power for financial crises, especially in more recent decades and in emerging market economies.

https://www.bis.org/publ/work566.htm

Source: Gertler and Hofmann (2016)

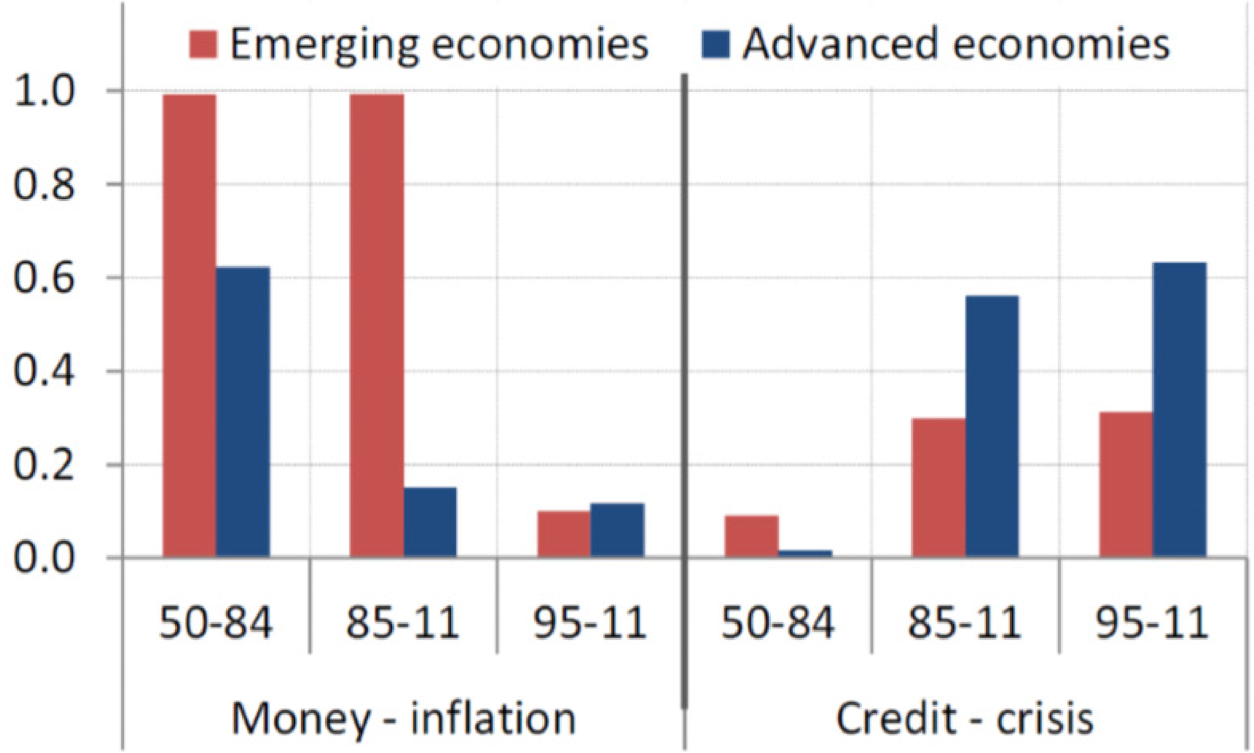


Figure 5. **Monetary facts in advanced and emerging economies over time.** Long-run impact of money growth on inflation and marginal effect of credit growth on crisis probability based on the estimates reported in Tables 5 and 6.