Monetary Macroeconomics

Lecture 11:
Savings and Investment

Dr. Pınar Yeşin

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University of Zurich

Chapter 16: Savings and Investment

Some questions we will try to answer:

- How are savings and investment determined in the economy?
- What is the effect of taxes on savings and consumption?
- What is the optimal level of capital in an economy?

The Model

OLG model with 2-period lived consumers.

When young, each has y_1 goods of endowment. When old, each has y_2 goods of endowment.

There is no fiat money in this economy.

Consumers can save for their old age. Gross real interest rate on savings is r.

The budget constraints for an agent born at t are:

When young

$$c_t^t + s_t \leq y_1$$

where c_t^t is consumption when young and s_t is savings.

When old

$$c_{t+1}^t \leq y_2 + rs_t$$

where c_{t+1}^t is consumption when old and rs_t is return on savings (principal and interest).

Thus

$$s_t \leq y_1 - c_t^t$$

$$c_{t+1}^t \leq y_2 + rs_t$$

$$c_{t+1}^t \leq y_2 + r[y_1 - c_t^t]$$

The life-time budget constraint is then

$$rc_t^t + c_{t+1}^t \leq ry_1 + y_2$$

in terms of future value.

And in terms of present value:

$$c_t^t + \frac{c_{t+1}^t}{r} \leq y_1 + \frac{y_2}{r}$$

Then the consumer's maximization problem is, given r:

$$\max_{c_t^t, c_{t+1}^t} u(c_t^t, c_{t+1}^t)$$
$$\text{s.t. } c_t^t + \frac{c_{t+1}^t}{r} \leq y_1 + \frac{y_2}{r}$$

Denote the solution (c_1^*, c_2^*). Then $s^* = y_1 - c_1^*$ is the amount saved.

s^* can be positive/zero/negative depending on the initial endowment, the size of the interest rate and the utility function.

Problem: Graph the consumer's problem and denote its solution on the graph.

Income vs. Wealth

There is a distinction between income and wealth:

Income is the current endowment, whereas wealth is the present value of life-time endowments.

Thus wealth of an agent born at t is

$$w_t = y_1 + \frac{y_2}{r}$$

Goods whose consumption rises with wealth are called normal goods. In our case, when w_t increases we expect both c_1^* and c_2^* to increase, because utility is assumed to be increasing in both.

Thus when wealth increases consumption in both periods of life increases.

Problem: Show the consumer's problem in a graph.

Note: Bus transportation, Mac&Cheese are examples of inferior goods. When your wealth increases you buy less of them, replacing them with car rides and steaks.

Consumption is determined by wealth. Savings adjust to income.

Let $r = 1.1$ and consider four cases:

y_1	y_2	$y_1 + \frac{y_2}{r}$	c_1^*	c_2^*	s^*
50	55	100	50	55	0
100	0	100	50	55	50
70	33	100	50	55	20
0	110	100	50	55	-50

In all the four cases above, wealth is constant. To consume at the optimum level according to consumer's maximization problem, the consumer has to save at different levels depending on the current income.

Effect of Taxes on Consumption and Saving

Assume that the consumers have to pay lump-sum taxes: τ_1 goods when young and τ_2 goods when old.

Then the budget constraints of an agent born in t are:

When young:

$$c_t^t + s_t \leq y_1 - \tau_1$$

When old

$$c_{t+1}^t \leq y_2 - \tau_2 + rs_t$$

And the life-time budget constraint is

$$c_t^t + \frac{c_{t+1}^t}{r} \le (y_1 - \tau_1) + \frac{(y_2 - \tau_2)}{r}$$

The right hand side is the present value of after-tax endowments.

Problem: Graph the consumer's maximization problem and denote its solution on the graph.

What is the Effect of a Wealth-Neutral Tax Change on Consumption and Savings?

Claim: An individual's consumption is determined by the present value of his lifetime taxes, but not on the timing of those taxes.

Proof: Consider the agent's life-time budget constraint with taxes. The right hand side of the equation will remain unchanged for all values of τ_1 and τ_2 such that $\tau_1 + \frac{\tau_2}{r}$ is constant.

If the budget constraint remains unchanged, the solution of the consumer's maximization problem stays unchanged too.

Hence the consumption does not change, and the savings adjust!

Thus a wealth-neutral tax change does not have an effect on consumption.

Savings adjust in order to keep the consumption at the same level as before the tax-change.

The Golden Rule Capital Stock

What is the optimal level of capital? Will a free market necessarily induce the optimal level of capital?

The Model

OLG model with 2-period lived identical consumers.

When young they have endowment y, and when old nothing.

There is no fiat money in this economy. The population grows at rate n.

Consumers can invest in capital: If k units are invested today, then next period it pays $f(k)$ back. The production function f has diminishing returns (i.e. f' is decreasing)

What is the feasibility constraint in the goods market?

$$N_t c_t^t + N_{t-1} c_t^{t-1} + N_t k_t \leq N_t y + N_{t-1} f(k_{t-1})$$

Consider stationary allocations:

$$N_t c_1 + N_{t-1} c_2 + N_t k \leq N_t y + N_{t-1} f(k)$$

$$c_1 + \frac{c_2}{n} + k \leq y + \frac{f(k)}{n}$$

Thus

$$c_1 + \frac{c_2}{n} \leq y + \Big(\frac{f(k)}{n} - k\Big)$$

The right hand side is the output net of the costs of investment per young person, i.e. net national product per young.

The optimal stationary capital stock maximizes the goods available for consumption in a stationary allocation.

$$\max_{0 \le k \le y} \frac{f(k)}{n} - k$$

The first order condition is

$$\frac{f'(k)}{n} - 1 = 0$$

Thus $f'(k) = n$ at the optimum, i.e. marginal product of capital equals to the economy's growth rate.

Problem: Graph the marginal benefit and marginal cost of capital. Denote their intersection k^*, the "*golden rule*".

Rate-of-return-equality implies that the real interest rates equal to the marginal product of capital.

So we can compare the real interest rates and the economy's growth rate to see if the economy is at the golden rule:

If $r > n$, then the economy has too little capital, there is capital underaccumulation.

If $r < n$, then the economy has too much capital, there is capital overaccumulation.

Nothing guarantees that a free market will yield to an interest rate that equals the economy's growth rate.

The capital stock is determined by the amount people wish to save for their future.

What can the government's do to reach the optimal capital accumulation in the economy?

1. Pay-as-you-go social security program

Tax the young, subsidize the old. This policy will reduce the income that can be saved, hence reduce the capital stock. Good to implement when capital is overaccumulated.

2. Issue government debt paying rate of return n

This will reduce the investment in capital when capital has lower rate of return. Good to implement when capital is overaccumulated.

Social Security

There are two basic ways to finance government social security payments to the old.

1. In a **fully funded pension plan**, the government taxes young workers and uses these contributions to purchase interest-bearing assets that will finance the pension payments to that same cohort of workers when they are old.

2. In a **pay-as-you-go plan**, the payments to the old are funded by taxes on those who are young in that period.

In a fully-funded pension plan, the payments of the young are saved at the market rate of return r and then used to finance the pensions of the same people who made the payments.

Although the contributions are mandatory, the people can still achieve their optimal consumption pattern by adjusting their private savings.

If both offer the same rate of return, the people really do not care which plan manages their pension funds; only the total savings matter.

On the other hand, in a pay-as-you-go system, the government undertakes no investment on behalf of the young and relies only on intergenerational transfers!

That means that the population growth rate and the rate of return on private savings play an important role in determining whether future generations like the pay-as-you-go system.

If $r > n$ then a person would be better off saving privately (because for each good given up today he would get r goods tomorrow by investing on his own, but only n goods through the social security system!)

So if $r > n$, the pension system *decreases* the wealth of future generations! Otherwise it increases.