Monetary Macroeconomics

Lecture 1:
A Simple Model of Money

Dr. Pınar Yeşin

February 26, 2021

University of Zurich

Introduction

What is the role of money in the economy?

Why do people use money?

We will try to answer these questions using a simple model.

What Kind of Model Shall We Use?

There are two possible frameworks in macroeconomic models:

- Overlapping generations models

  Introduced by Paul Samuelson in 1958

  "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money," Journal of Political Economy Vol. 66.

- Models with infinitely-lived households

We will focus on the first framework in this course.

Overlapping Generations Model (OLG)

Agents live for two periods.

Every period a new generation is born — they are the young in that period.

And next period they get old — at the end of that period they die.

period 1: there are N_0 initial old, and N_1 young people

period 2: there are N_1 old, and N_2 young people

. .

period t: there are N_{t-1} old, and N_t young people.

There is one consumption good.

Agents have y units of endowment of the consumption good when they are young, and 0 units when they are old.

The consumption good is perishable.

The preferences of an agent born at time t over consumption when young and old are given by the utility function

$$u_t(c_t^t, c_{t+1}^t)$$

where c_t^t is his consumption when young, and c_{t+1}^t is his consumption when old.

We assume that the utility function has all the "nice" properties, i.e. increasing in both c_t^t and c_{t+1}^t, and concave, etc.

Utility of the initial old household is given by

$$u_0(c_1^0)$$

What are the Feasible Allocations?

In any period t, the total amount of consumption in the economy should be less than or equal to the amount of goods available.

$$N_t c_t^t + N_{t-1} c_t^{t-1} \leq N_t y$$

The first term on the left hand side is the total consumption of the young people, and the second term on the left hand side is the total consumption of the old people (they were born in period $t-1$!)

The right hand side is the total number of available goods = total endowment of the young, since the old do not have endowment.

This is the **feasibility condition**.

What is a Competitive Equilibrium?

A competitive equilibrium is **prices and allocations** such that

- Given prices, the allocations are such that they maximize the agents' utilities

- Markets clear, i.e. allocations are feasible.

What is the Competitive Equilibrium in this Economy?

There is no opportunity to trade between the agents.

The initial old does not have anything to offer to buy consumption good from the young, so he starves.

The initial young would love to write a contract with next period's young so that he can consume something when old, but next period's young is not born yet!

And the people of the same generation are in the exact same situation, there is no point in trading with somebody of their generation.

The unique competitive equilibrium is **AUTARKY**.

Every agent consumes his own endowment when young and starves when old! There is no trade.

What is the Golden Rule Allocation in this Economy?

The golden rule allocation solves the **social planner's problem**.

That is it maximizes the utility of all *future young* generations subject to the feasibility constraint.

Some Simplifying Assumptions

1. Assume that there is no population growth, i.e. $N_t = N$ is constant for all t.

So the feasibility constraint becomes

$$N_t c_t^t + N_{t-1} c_t^{t-1} \leq N_t y$$

$$N c_t^t + N c_t^{t-1} \leq N y$$

$$c_t^t + c_t^{t-1} \leq y$$

2. Moreover, consider only the allocations that are time independent (stationary) — Nothing is different among the generations in a fundamental way anyway! So why treat them differently?

i.e. let $c_t^t = c_1$ and $c_t^{t-1} = c_2$ for all t.

That is everybody consumes c_1 when young and c_2 when old. Consumption does not depend on which generation the household belongs to.

Hence the feasibility constraint becomes

$$c_1 + c_2 \leq y$$

Note: c_1 does not have to be equal to c_2!

What is the Golden Rule Allocation now?

The social planner's problem is

$$\max u(c_1, c_2)$$
$$\text{subject to } c_1 + c_2 \leq y$$

Denote the solution to this problem as (c_1^*, c_2^*). This allocation maximizes the utility of *all the future generations* subject to the feasibility constraint.

Note that it does *not* maximize the initial old's utility — he only cares about consumption when old, he will not be young again.

Hence the golden rule only considers the welfare of the future generations — which makes sense since there is only one initial old generation but there are infinitely many future generations!

Can the golden rule allocation be achieved in this economy?

No. The golden rule allocation cannot be achieved in competitive equilibrium in this economy — unless people do not care about consumption when old, which we ruled out before.

Problem:

Consider the overlapping generations model above when

$y = 9$, $N_t = 1$, and

$u_t(c_t^t, c_{t+1}^t) = \ln c_t^t + 0.8 \ln c_{t+1}^t$.

Find the competitive equilibrium and the stationary golden rule allocation.

Fiat money

Assume that there is a good that can be stored costlessly by the agents but it cannot be produced or consumed. Agents can exchange it but it is *intrinsically useless*. It is not perishable!

We call this **fiat money**.

A **monetary equilibrium** is a competitive equilibrium where fiat money is valued.

That is the agents trade fiat money for consumption good.

Note: For fiat money to be valued, the money supply must be limited and it must be impossible to counterfeit.

Introduce Fiat Money to this Model

At period t, generation t is born with y units of initial endowment of the consumption good and no fiat money.

He can trade some of his endowment for money so that he can buy some consumption good when he is old.

His budget constraint in period t in money terms is

$$p_t c_t^t + m_t \leq p_t y$$

The first term is the monetary value of his consumption when young, m_t is the money he acquires when young, and the right hand side is the monetary value of his endowment when young.

His budget constraint next period $(t+1)$ in money terms is

$$p_{t+1}c_{t+1}^{t} \leq m_t$$

The left hand side is the monetary value of his consumption when old, and the right hand side is the money he acquired last period.

We can combine the two budget constraints and obtain the *life-time budget constraint* of generation t.

$$p_t c_t^t + p_{t+1}c_{t+1}^{t} \leq p_t y$$

The left hand side is the value of his total consumption and the right hand side is his initial wealth.

Lifetime Budget Constraint and Prices

$$p_t c_t^t + p_{t+1} c_{t+1}^t \leq p_t y$$

$$c_t^t + \frac{p_{t+1}}{p_t} c_{t+1}^t \leq y$$

Recall that in a competitive equilibrium the prices are given — the agents do not have any control on prices.

$$\frac{p_{t+1}}{p_t} = 1 + \pi_t$$

π is denotes the inflation rate.

How do We Find the Prices?

Assume that there is M_t units of fiat money in the economy in period t.

In equilibrium money supply must equal to money demand!

Who demands money in this economy? The young only.

How much?

From his budget constraint when young

$$m_t = p_t y - p_t c_t^t$$

So the total money demand is

$$M_t = (p_t y - p_t c_t^t) N_t$$

Thus

$$M_t = p_t N_t (y - c_t^t)$$

and the price level is given by

$$p_t = \frac{M_t}{N_t(y - c_t^t)}$$

$$p_{t+1} = \frac{M_{t+1}}{N_{t+1}(y - c_{t+1}^{t+1})}$$

Therefore the rate at which the price changes is

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{N_{t+1}(y - c_{t+1}^{t+1})} \frac{N_t(y - c_t^t)}{M_t}$$

Stationary Monetary Equilibrium with Constant Population

Assume that $N_t = N$ for all t.

Consider the stationary allocations.

So $c_t^t = c_1$ and $c_{t+1}^t = c_2$ for all t.

Thus

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{N_{t+1}(y - c_{t+1}^{t+1})} \frac{N_t(y - c_t^t)}{M_t}$$

becomes

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{M_t}$$

When the Money Supply is Constant

Then

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{M_t} = 1$$

That is the prices are also *constant*!

Then the generation t's budget constraint reduces from

$$c_t^t + \frac{p_{t+1}}{p_t} c_{t+1}^t \leq y$$

to

$$c_1 + c_2 \leq y$$

So his maximization problem is

$$\max\ u(c_1, c_2)$$
$$\text{subject to } c_1 + c_2 \leq y$$

Does this look familiar???

The generation t's maximization problem *is identical* to the social planner's problem that maximizes future generations' utility.

The stationary monetary equilibrium is the golden rule allocation!

By introducing fiat money in the economy the welfare of the individuals in the economy are improved.

The competitive equilibrium is not autarky anymore. The young and the old can trade with each other and everyone is better off!

We will continue from here in the next lecture ...