Monetary Macroeconomics

Lecture 4:

Inflation

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Chapter 4: Inflation

What if the Fiat Money Stock is Growing?

In chapters 1 and 2 we developed a simple model to analyze the role of money in the economy.

We had assumed that the fiat money stock was constant.

In this chapter we will relax this assumption and analyze the effects of *increasing fiat money stock* on the economy.

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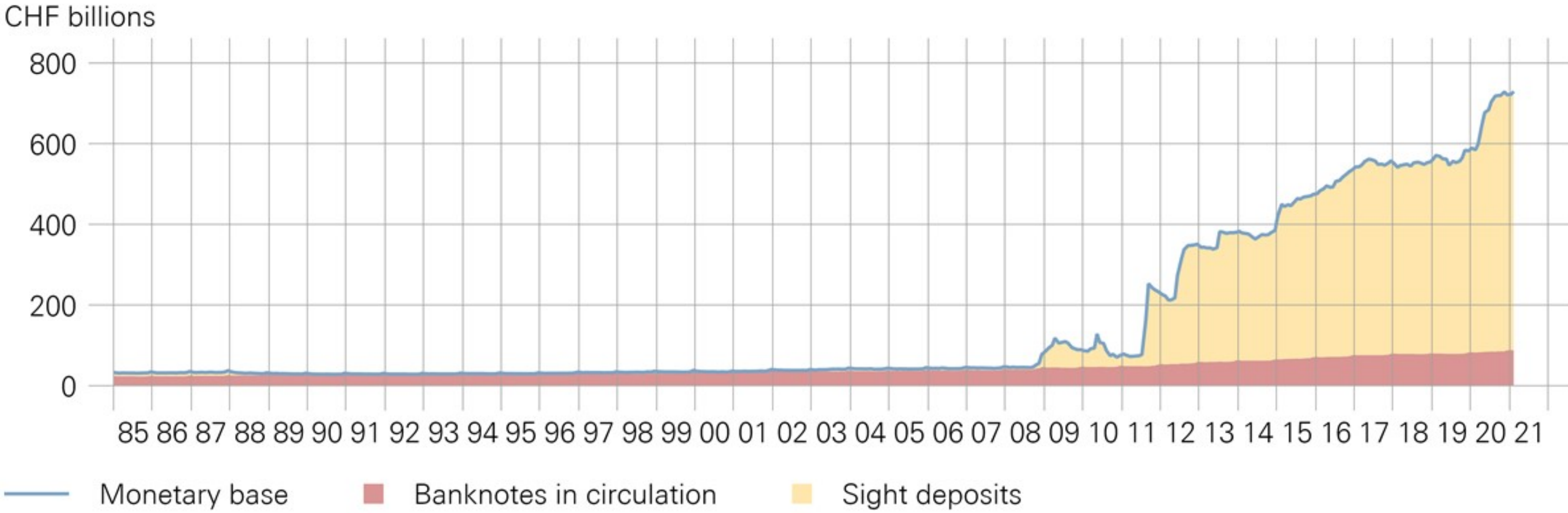
MONETARY BASE



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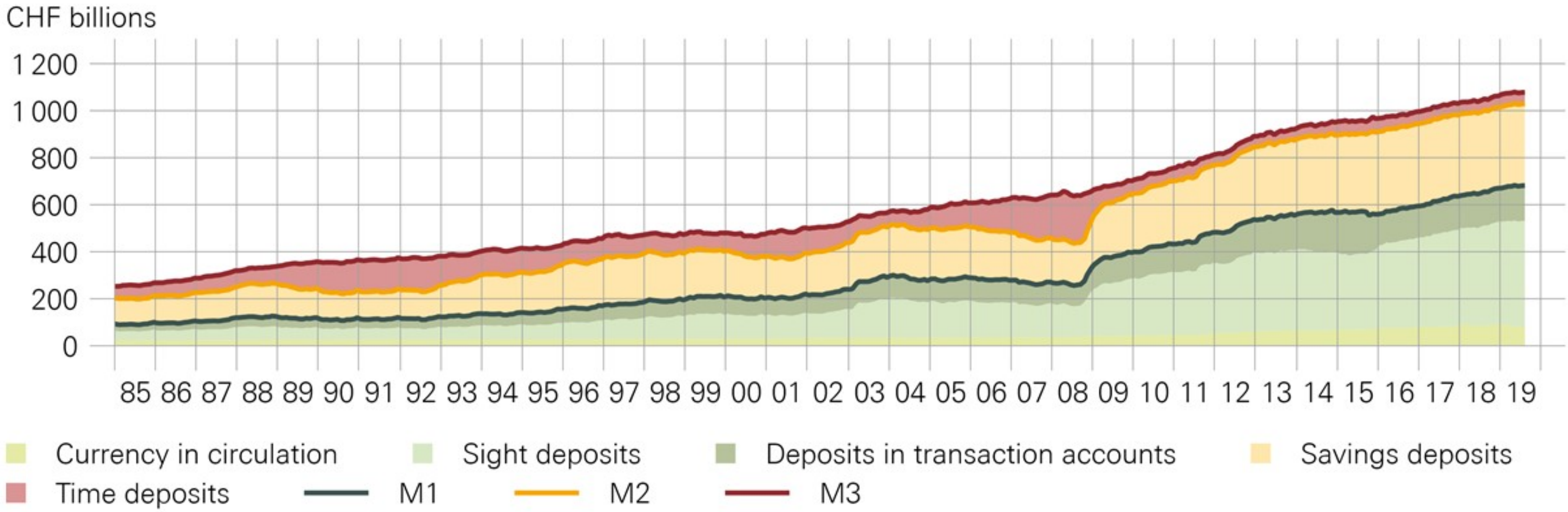
MONETARY AGGREGATES



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The Model

Overlapping generations model as in Chapter 1.

One perishable consumption good.

Agents live for two periods; when young they have y units of the consumption good as endowment, and when old they have nothing.

There is fiat money in this economy that can be used for trade.

The Government

The money stock grows at rate z each period.

$$M_t = zM_{t-1} \quad \text{for all } t \quad , \quad z > 1$$

The money supply increases because the government prints new fiat money every period,

$$M_t - M_{t-1} = M_t - \frac{1}{z}M_t = \Big(1 - \frac{1}{z}\Big)M_t$$

and gives away the new printed money to the old as lump-sum transfers (i.e. the amount given to a person does not depend on any decision made by that old person).

The **government's budget constraint** in period t is

$$\left(1 - \frac{1}{z}\right)M_t = N_{t-1}T_t$$

The left hand side is the "income" of the government, and the right hand side is the "expenditure" of the government.

T_t is the lump-sum transfer to the old in period t.

Generation t's Budget Constraints

When young,

$$p_t c_t^t + m_t \leq p_t y$$

so that consumption when young plus acquired money holdings is less than the value of his endowment.

When old,

$$p_{t+1} c_{t+1}^t \leq m_t + T_{t+1}$$

so that consumption when old is less than his money holdings plus the transfer from the government.

Hence his life-time budget constraint is

$$p_t c_t^t + p_{t+1} c_{t+1}^t \leq p_t y + T_{t+1}$$
$$c_t^t + \frac{p_{t+1}}{p_t} c_{t+1}^t \leq y + \frac{T_{t+1}}{p_t}$$

Feasibility in the Money Market

Feasibility in the money market requires that money demand equals to money supply.

$$N_t p_t (y - c_t^t) = M_t$$

Thus $p_t = \frac{M_t}{N_t (y - c_t^t)}$ as before.

So $\frac{p_{t+1}}{p_t} = \frac{M_{t+1} N_t (y - c_t^t)}{N_{t+1}(y - c_{t+1}^{t+1}) M_t}$.

Constant Population and Stationary Economy

Assume that $N_t = N$, and that $c_t^t = c_1$ and $c_{t+1}^t = c_2$ for all t.

Hence

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1} N_t (y - c_t^t)}{N_{t+1}(y - c_{t+1}^{t+1}) M_t} = z$$

Since $z > 1$, the prices are increasing over time. There is inflation!

$$p_{t+1} = z p_t$$

Note that the prices are increasing at the *same rate* as money supply is increasing!

Thus generation t's life-time budget constraint becomes

$$c_1 + zc_2 \leq y + \frac{T_{t+1}}{p_t}$$

or equivalently

$$c_1 + zc_2 \leq y + \frac{p_{t+1}}{p_t}\frac{T_{t+1}}{p_{t+1}}$$

$$c_1 + zc_2 \leq y + z\frac{T_{t+1}}{p_{t+1}}$$

Claim: $\frac{T_{t+1}}{p_{t+1}}$ is constant for all t in a stationary equilibria.

Intuitively, the real value of the transfer from the government should not be changing over time if the allocation is stationary.

But we have to prove that!

Claim: $\frac{T_{t+1}}{p_{t+1}}$ is constant for all t in a stationary equilibria.

Proof:

Government's budget constraint was

$$\left(1 - \frac{1}{z}\right) M_t = N_{t-1} T_t$$

So

$$T_t = \frac{(1 - \frac{1}{z}) M_t}{N_{t-1}}$$
$$T_{t+1} = \frac{(z - 1) M_{t+1}}{z N_t}$$

And from the feasibility in the money market we got

$$p_{t+1} = \frac{M_{t+1}}{N_{t+1}(y - c_1)}$$

Thus the real value of the transfer in period $t+1$ is

$$\frac{T_{t+1}}{p_{t+1}} = \frac{\frac{(z-1)M_{t+1}}{zN_t}}{\frac{M_{t+1}}{N_{t+1}(y-c_1)}} = \frac{(z-1)(y-c_1)}{z}$$

Note that this term is constant over time!

Hence the buying power of the transfer does not change over time.

Denote $\frac{T_t}{p_t} = a$.

Generation t's life-time budget constraint reduces to

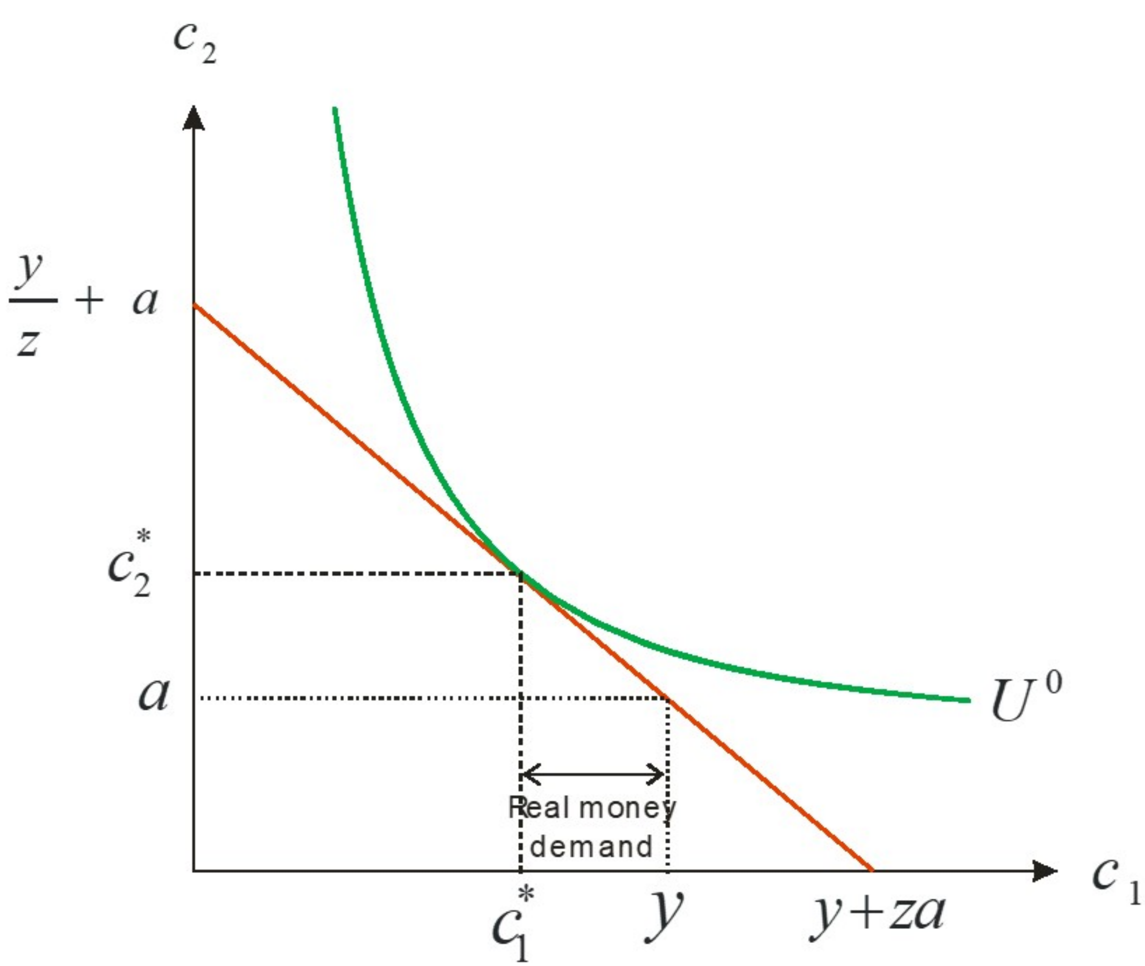
$$c_1 + zc_2 \leq y + z\frac{T_{t+1}}{p_{t+1}}$$
$$c_1 + zc_2 \leq y + za$$

Thus his maximization problem is

$$\begin{aligned} &\text{max} \quad u(c_1, c_2) \\ &\text{s.t. } c_1 + zc_2 \leq y + za \end{aligned}$$

Denote the solution as (c_1^*, c_2^*).

Problem: Graph the monetary equilibrium.



c_2
$\frac{y}{z} + a$
c_2^*
a
U^0
Real money demand
c_1^*
y
$y+za$
c_1

Golden Rule Allocation

Recall that the feasibility constraint in the goods market with a constant population is

$$c_1 + c_2 \leq y$$

Hence the social planner's problem that maximizes the utility of all future young generations is

$$\begin{aligned} \text{max} \quad & u(c_1, c_2) \\ \text{s.t.} \ & c_1 + c_2 \leq y \end{aligned}$$

The solution will be different from (c_1^*, c_2^*)! Since the subject to constraints of the two problems differ.

Denote the solution to the social planner's problem as $(\hat{c}_1, \hat{c}_2)$.

Problem: Graph the golden rule allocation.

Claim: (c_1^*, c_2^*) lies on the line $c_1 + c_2 \leq y$. (The monetary equilibrium is at the intersection of the budget constraint and feasibility condition!)

Proof: Since (c_1^*, c_2^*) solves the generation t's maximization problem, it satisfies his budget constraint:

$$c_1^* + zc_2^* = y + za$$

We also know that at the equilibrium

$$a = \frac{(z-1)(y-c_1^*)}{z}$$

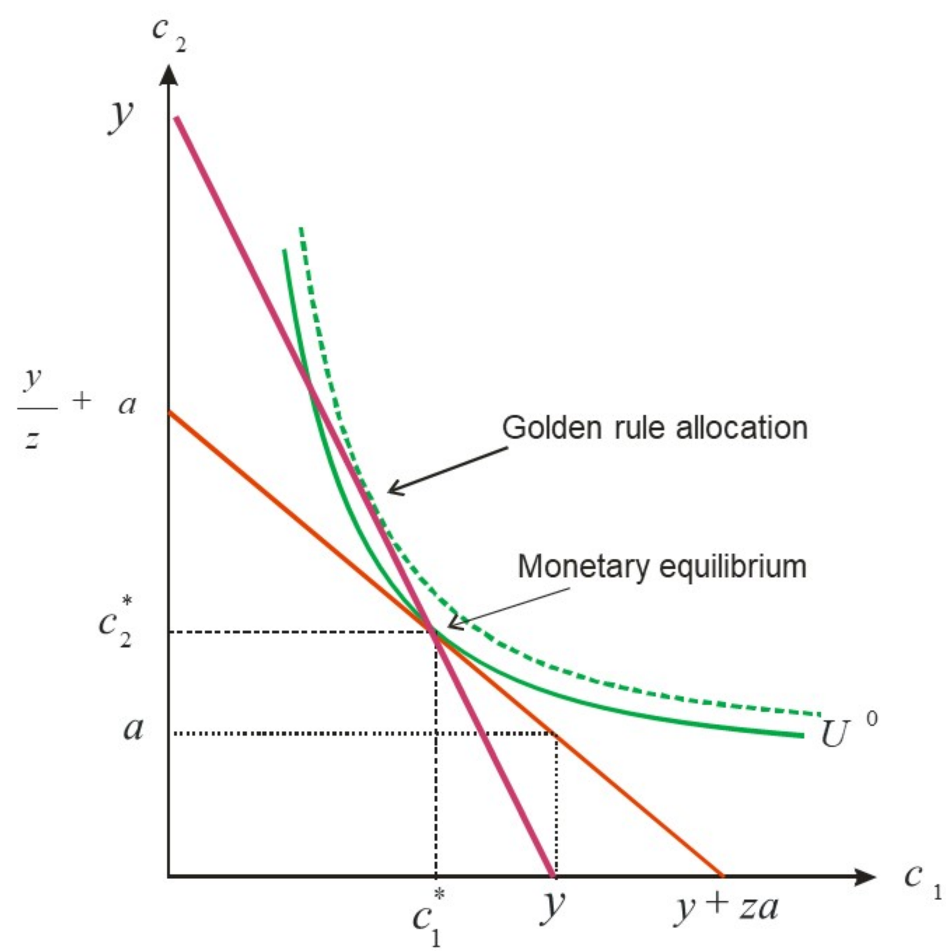
Thus

$$
\begin{aligned}
c_1^* + zc_2^* &= y + za \\
c_1^* + zc_2^* &= y + (z-1)(y - c_1^*) \\
c_1^* + zc_2^* &= y + zy - zc_1^* - y + c_1^* \\
zc_1^* + zc_2^* &= zy \\
c_1^* + c_2^* &= y
\end{aligned}
$$

Hence (c_1^*, c_2^*) lies on the line $c_1 + c_2 \leq y$.

Problem

Graph the monetary equilibrium and the golden rule allocations and show that the monetary equilibrium with an expanding money supply is *inefficient*!


c_2
y
$\frac{y}{z} + a$
Golden rule allocation
Monetary equilibrium
c_2^*
a
U^0
c_1
c_1^*
y
$y + za$

Intuition Why Monetary Equilibrium is Inefficient

Note that $c_1^* > \hat{c}_1$ and $c_2^* < \hat{c}_2$.

When there is inflation, money loses its value over time. So people prefer to hold less money, i.e. they consume more in the first period and thus their real money balances are lower.

So people economize on their money holdings due to inflation. They cannot take full advantage of fiat money benefits next period.

Money supply expansion is like an implicit taxation on money holdings.

Problem

Show that the stationary monetary equilibrium with a growing money supply is still inefficient when the **population is growing**.

Assume that $N_t = nN_{t-1}$ where $n > 1$. And that $M_t = zM_{t-1}$ where $z > 1$, and that the government distributes transfers to the old with the new printed money.

Solution

We first write down how the prices change in a stationary economy:

$$\begin{aligned}\frac{p_{t+1}}{p_t} &= \frac{M_{t+1}}{N_{t+1}(y-c_1)}\frac{N_t(y-c_1)}{M_t} \\ &= \frac{zM_t}{nN_t(y-c_1)}\frac{N_t(y-c_1)}{M_t} \\ &= \frac{z}{n}\end{aligned}$$

Hence $p_{t+1} = \frac{z}{n} p_t$.

If $z > n$, then there is inflation.

If $z = n$, then the prices are constant over time.

If $z < n$, then there is deflation.

Recall that the real value of the transfer is constant over time in a stationary economy. Denote it again with a.

Thus the maximization problem of an agent born in t is

$$\begin{aligned} \text{max} \quad & u(c_1, c_2) \\ \text{s.t.} \; & c_1 + \frac{z}{n} c_2 \leq y + \frac{z}{n} a \end{aligned}$$

But the feasibility in the goods market with a growing population is

$$
\begin{aligned}
N_t c_1 + N_{t-1} c_2 &\leq N_t y \\
n N_{t-1} c_1 + N_{t-1} c_2 &\leq n N_{t-1} y \\
c_1 + \frac{1}{n} c_2 &\leq y
\end{aligned}
$$

Thus the social planner's problem is

$$
\begin{aligned}
\max \quad & u(c_1, c_2) \\
\text{s.t.} \quad & c_1 + \frac{1}{n} c_2 \leq y
\end{aligned}
$$

Since the two problems do not coincide; the monetary equilibrium cannot achieve the golden rule allocation.

Result

Expansion of money supply yields to inefficiency when the new printed money is used for transfers to the old.

Only when $z = 1$, a will equal to zero, and the two problems will be identical! But this corresponds to a constant money stock!

Money supply expansion for transfers to the old results in an **inefficient** monetary equilibrium.

Seigniorage Revenue

Real seigniorage revenue is defined as how many units of consumption good the new printed money worth is.

$$\text{Real seigniorage revenue } = \frac{M_t - M_{t-1}}{p_t}$$

In our model above, $M_t = zM_{t-1}$. Hence

$$\text{seigniorage } = \left(1 - \frac{1}{z}\right)\frac{M_t}{p_t}$$

The second term on the right hand side, $\frac{M_t}{p_t}$, can be *considered* as the seigniorage tax base (the number of goods that the current money supply can buy).

And the first term, $\left(1 - \frac{1}{z}\right)$, can be *considered* as the seigniorage tax rate (it will be between 0 and 1 for $z \geq 1$).

It is believed that the real seigniorage revenue as a function of z (i.e. rate of increase in money supply) is first increasing and then reaches a maximum and then starts decreasing.

This means that a government cannot increase its seigniorage revenue forever by increasing the rate at which it is printing money (similar to the idea of Laffer curve for income tax revenue).

For cross-country comparison, one usually scales the seigniorage revenue either using the size of the government or the size of the output in that country.

$$\text{Seigniorage } = \frac{\frac{M_t - M_{t-1}}{p_t}}{\frac{E_t}{p_t}} = \frac{M_t - M_{t-1}}{E_t}$$

where E_t is the nominal government expenditures.

So seigniorage is expressed as the fraction of the government expenditures that could be financed through new money printing.

Alternatively,

$$\text{Seigniorage } = \frac{\frac{M_t - M_{t-1}}{p_t}}{\frac{Y_t}{p_t}} = \frac{M_t - M_{t-1}}{Y_t}$$

where Y_t is the nominal output.

So seigniorage is expressed as the fraction of total output that could be bought with the new printed money.

Average Interest Rates and Seigniorage Revenues across countries (1976-2000)

Country	Interest Rate (%)	Seigniorage ($\Delta M/E$) (%)	Seigniorage ($\Delta M/GDP$) (%)
Australia	11.54	1.61	0.36
Bolivia	54.03	13.81	3.14
Canada	8.86	1.08	0.25
Chile	39.86	28.67	7.53
Colombia	31.78	5.99	1.99
Denmark	7.20	2.04	0.62
Egypt	12.63	11.82	4.80
Greece	17.66	4.54	1.35
India	10.04	12.57	1.89
Israel	113.47	16.49	11.62
Italy	12.22	1.73	0.59
Malaysia	5.03	8.73	2.07
Mexico	30.80	11.48	2.90
Norway	9.11	1.09	0.38
Peru	103.97	29.29	4.78
Portugal	14.69	2.18	0.85
Spain	10.80	4.84	0.96
South Korea	7.56	6.26	0.92
Sweden	7.40	1.03	0.43
Switzerland	3.31	1.84	0.19
Turkey	43.17	15.18	3.14
Uruguay	88.10	20.14	4.95
U.S.	6.67	1.26	0.39

Calculations are based on the IMF IFS (International Financial Statistics

From this table you can already see that higher interest rates (i.e. higher inflation rates) do not generally yield to higher seigniorage revenue! See, for example, Peru.