

Answers to Quiz #1

Consider a 2-period overlapping generations model with the following characteristics:

There is only one type of perishable consumption good in the economy. Each agent has y units of initial endowment of the consumption good when young and nothing when old.

The agents' preferences are given by the utility function $u(c_t^t, c_{t+1}^t)$ where c_t^t denotes consumption when young and c_{t+1}^t denotes consumption when old.

The population is growing at rate n , i.e. $N_t = n * N_{t-1}$ agents are born in each period t such that $n > 1$.

The only asset to save for tomorrow is fiat money.

Each period the government **has to finance** an aggregate government spending for a new Gotthard Tunnel, amounting to g **goods per young person**.

In order to pay for its spending, the government prints fiat money at rate $z > 1$ every period. $M_t = z * M_{t-1}$ for every period t . That is z is such that the new printed money can finance the aggregate government spending in each period.

(1.) What is the feasibility constraint in this economy?

$$N_t c_t^t + N_{t-1} c_t^{t-1} + N_t g \leq N_t y$$

Because the population grows at the constant rate n , the feasibility constraint can be written as:

$$c_t^t + \frac{1}{n} c_t^{t-1} \leq y - g$$

Note that the government consumption should be included in the feasibility constraint!

(2.) What is the social planner's problem? Show the golden rule allocation in a figure.

We write the social planner's problem (maximize future generations' utility subject to the feasibility constraint in the goods market in a stationary economy):

$$\begin{aligned} & \max U(c_1, c_2) \\ & \text{s.t. } c_1 + \frac{1}{n} c_2 \leq y - g \end{aligned}$$

The figure is depicted under the answer of (5.)

(3.) What is the agent's budget constraint when young and when old? Write down the lifetime budget constraint.

The budget constraint of a representative agent when young is

$$p_t c_t^t + m_t \leq p_t y$$

and when old his budget constraint is

$$p_{t+1} c_{t+1}^t \leq m_t$$

Therefore his lifetime budget constraint is

$$\begin{aligned} p_t c_t^t + p_{t+1} c_{t+1}^t &\leq p_t y \\ c_t^t + \frac{p_{t+1}}{p_t} c_{t+1}^t &\leq y \end{aligned}$$

Note that there are no taxes or transfers! Money is the only asset to save in for tomorrow!

(4.) What is the government's budget constraint?

The government's budget constraint sets government expenditures to build the tunnel equal to government revenue from money printing.

$$\begin{aligned} p_t N_t g &= M_t - M_{t-1} \\ p_t N_t g &= \left(1 - \frac{1}{z}\right) M_t \end{aligned}$$

The government prints fiat money at rate z , such that the above constraint is satisfied. Note that there are no other sources of revenue and no other government expenses!

(5.) Show the monetary equilibrium in a figure.

We find the ratio of the prices in a stationary economy using the feasibility constraint in the money market:

$$\begin{aligned} p_t &= \frac{M_t}{N_t(y - c_t^t)} \\ \frac{p_{t+1}}{p_t} &= \frac{\frac{M_{t+1}}{N_{t+1}(y - c_{t+1}^{t+1})}}{\frac{M_t}{N_t(y - c_t^t)}} = \frac{N_t}{N_{t+1}} \frac{M_{t+1}}{M_t} \frac{(y - c_t^t)}{(y - c_{t+1}^{t+1})} \end{aligned}$$

Because the fiat money stock and the population are both growing, prices evolve based on the values of z and n .

$$\frac{p_{t+1}}{p_t} = \frac{z}{n}$$

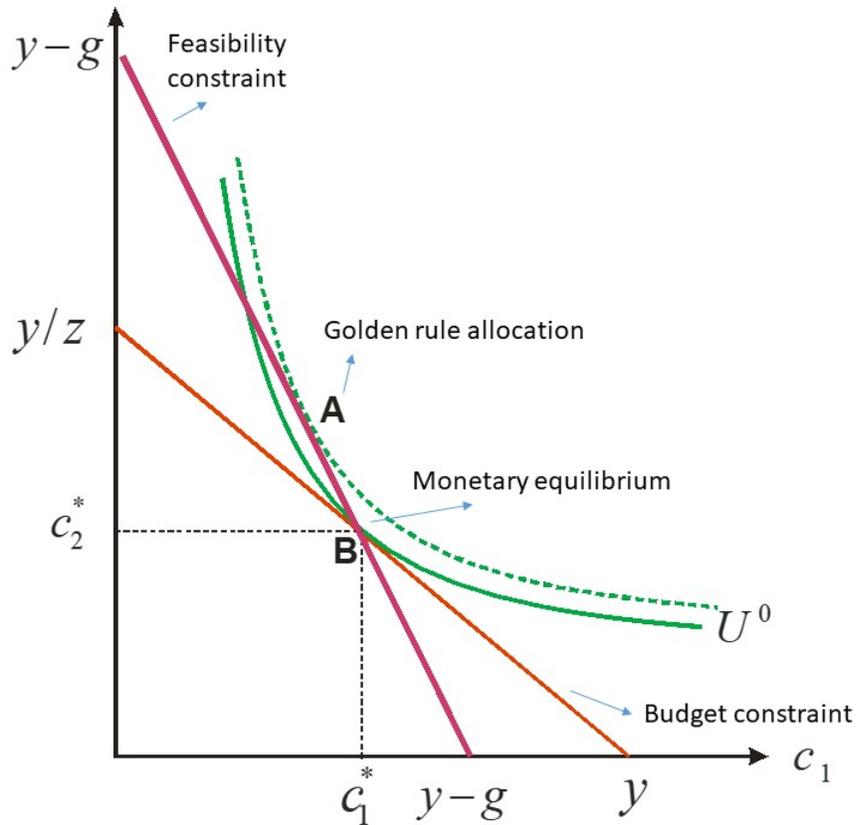
Therefore, the consumer's life-time budget constraint in a stationary economy becomes

$$c_1 + \frac{z}{n} c_2 \leq y$$

And the consumer solves the following maximization problem:

$$\begin{aligned} \max U(c_1, c_2) \\ \text{s.t. } c_1 + \frac{z}{n} c_2 &\leq y \end{aligned}$$

The solution to this problem is the stationary monetary equilibrium.



(6.) Is the monetary equilibrium efficient? Why?

No, the monetary equilibrium is not efficient, because the consumer's maximization problem is not the same as the social planner's problem.

In particular, printing money distorts the consumers' decision on how much to save for tomorrow and the golden rule allocation cannot be achieved in a monetary equilibrium if the government prints money.

A better way to finance the government spending would be through lump-sum taxes.